$$\begin{array}{l} 1,\ 3,\ 5,\ \ldots.\ 2r-1 \\ 2r+1,\ 2r+3,\ \ldots.\ 4r-1 \\ 4r+1,\ 4r+3,\ \ldots.\ 6r-1 \\ \vdots \qquad \vdots \qquad \vdots \\ 2(n-1)r+1,\ \ldots.\ 2nr-1 \end{array}$$

The sum of the terms in the nth group is

$$\frac{r}{2}(\text{1st term}+\text{last term})=\frac{r}{2}(4rn-2r)=r^2(2n-1).$$

Solved similarly by *G. B. M. ZERR*, *J. H. DRUMMOND*, and *J. SCHEFFER*.

147. Proposed by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Stroud, Gloucestershire. England.

Prove that $x=a^x$ has never more than two real roots, and find the condition for no real roots.

No solution of this problem has been received.

148. Proposed by R. D. BOHANNAN, Ph. D., Professor of Mathematics, Ohio State University, Columbus, O.

If $\frac{x}{a+\alpha}+\frac{y}{b+\beta}+\frac{z}{c+\gamma}=1$, $\frac{x}{a+\beta}+\frac{y}{b+\beta}+\frac{z}{c+\beta}=1$, $\frac{x}{a+\gamma}+\frac{y}{b+\gamma}+\frac{z}{c+\gamma}=1$, show, without solving, that $x+y+z=a+\alpha+b+\beta+c+\gamma$.

No solution of this problem has been received.

149. Proposed by JOSEPH V. COLLINS, Ph. D., Stevens Point, Wis.

1. How many different football elevens can be sent out from a school having twenty players? In how many ways can eleven men line up?

Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

It is possible to send out $\frac{20!}{11!\ 9!}$ elevens, or 167960 elevens.

The eleven men can line up 11! ways.

Also solved by *G. B. M. ZERR* and *C. A. LINDEMANN*.

150. Proposed by JOSEPH V. COLLINS, Ph. D., Stevens Point, Wis.

2. How many sets of officers (president, vice-president, treasurer, and secretary) can a society of forty persons elect? How many committees of four persons, supposing no attention is paid to positions on the committees? How many committees in which the chairman is selected?

Solution by P. H. PHILBRICK, C. E., Lake Charles, La., and C. A. LINDEMAN, Professor of Mathematics Virginia Union University, Richmond, Va.

The society can elect $\frac{40!}{36!.4!}$ sets of officers.

The number of committees, no attention being paid to positions on the

same, is also $\frac{40!}{36!.4!}=\frac{40.39.38.37}{1.2.3.4}$.

The number of committees in which the chairman is selected, leaving 39 from whom to choose, is $\frac{39!}{36!.3!}=\frac{39.38.37}{6}=9139$.

Also solved by *G. B. M. ZERR.*

151. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Solve the equations:

$$\begin{aligned}
&x+y+z+u+w=1.\\
&ax+by+cz+du+ew=h,\\
&a^2x+b^2y+c^2z+d^2u+e^2w=h^2,\\
&a^3x+b^3y+c^3z+d^3u+e^3w=h^3,\\
&a^4x+b^4y+c^4z+d^4u+e^4w=h^4.
\end{aligned}$$

Solution by CLARENCE E. COMSTOCK, Professor of Mathematics. Bradley Polytechnic Institute. Peoria. Ill.

Solution by determinants.

$$x=\begin{vmatrix} 1 & 1 & 1 & 1 & 1\\ h & b & c & d & e\\ h^2 & b^2 & c^2 & d^2 & e^2\\ h^3 & b^3 & c^3 & d^3 & e^3\\ h^4 & b^4 & c^4 & d^4 & e^4\end{vmatrix} \div \begin{vmatrix} 1 & 1 & 1 & 1 & 1\\ a & b & c & d & e\\ a^2 & b^2 & c^2 & d^2 & e^2\\ a^3 & b^3 & c^3 & d^3 & e^3\\ a^4 & b^4 & c^4 & d^4 & e^4\end{vmatrix} \equiv \frac{\Delta_{a=b}}{\Delta}.$$

By the factor theorem, we get

$$\Delta=(a-b)(a-c)(a-d)(a-e)(b-c)(b-d)(b-e)(c-d)(c-e)(d-e).$$

$\Delta_{a=h}$=the same with a replaced by h.

$$\therefore x=\frac{(h-b)(h-c)(h-d)(h-e)}{(a-b)(a-c)(a-d)(a-e)}.$$

Since a, b, c, d, e appear in the same way, the principle of symmetry enables us to write the values y, z, u, w at once.

$$y=\frac{(h-a)(h-c)(h-d)(h-e)}{(b-a)(b-c)(b-d)(b-e)},\ z=\frac{(a-h)(c-h)(d-h)(e-h)}{(b-a)(c-a)(d-a)(e-a)},$$

$$u=\frac{(a-h)(b-h)(c-h)(e-h)}{(a-d)(b-d)(c-d)(e-d)},\ \text{and}\ w=\frac{(a-h)(b-h)(c-h)(d-h)}{(a-e)(b-e)(c-e)(d-e)}$$

Solved in a similar manner by *G. B. M. ZERR.*

152. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Solve by a short original method, if possible: